Developing and Assessing Mathematical Thinking and Reasoning

The focal points of this entry are two mathematical concepts from statistics: experimental design/experiment simulation and linear regression. For experimental design, some of the important elements students should know are the three principles of experimental design:

1. Control – needed to counter effects of lurking variables, etc. The simplest form of control is comparison. Experiments should compare two or more methods in order to avoid confounding the results of one method with other influence.
2. Randomization – subjects (people, animals, objects) assigned to methods by pure chance. This creates groups that are similar, except for chance variation. For example, a table of random digits or random number generators can be used to select subjects.
3. Replication – perform experiment on many subjects, or repeat the experiment with same population of subjects (but a separate randomly selected sample), to reduce chance variation in the results.

When simulating an experiment, using manipulatives or computer algorithms in place of actual subjects, students must know that these same three principles must apply and must be accounted for in the design of the simulation. For linear regression, some of the important elements students should learn are:

1. Scatter plots – an initial plotting of the data to determine if linear modeling may be possible.
2. Various methods for linear regression – a study of such techniques as least squares and median fit and the differences between the techniques.

3. The meaning of linear regression results – how to determine if a linear model that has been constructed is a good model.

I have chosen these concepts to demonstrate two important pedagogical techniques I have learned: 1) thematic instruction, and 2) problem-based learning.

Thematic instruction involves the use of a common and recurring *theme* in the instructional materials being used. This helps to reinforce the idea with students that mathematics doesn't occur in a vacuum and that math isn't just an endless series of unrelated, standalone equations to be worked and word problems to be solved. The goal is to help students make connections between mathematical concepts and continually build upon their mathematical foundation by having a common scenario against which their mathematical knowledge and reasoning will be applied. In this particular case, the common theme is a pond located on a large tract of land in a rural community. In the two specific lessons presented here, the pond is utilized as a catalyst for designing experiments (to count how many fish are in the pond) and studying linear models (to find out why the fish population is declining). In the context of an entire statistics course, I would find ways to incorporate this pond into all other concepts, too. The central theme should increase the students' engagement levels, since they will begin to think "What's going to happen with our pond next?" and be eager to apply what they've learned to solving whatever problem happens next.

Problem-based learning, then, is the real reason for pursuing the thematic instruction. Current textbooks and other curriculum materials do a good job of helping to teach the facts and mechanics of various math concepts. However, as pointed out in the paper I included for Entry 3, the development of a deep understanding of mathematical concepts requires going beyond traditional lecture-based learning techniques, and problem-based learning is one of the "power tools" available to educators to help reach that goal. Most current curriculum materials don't provide much help with problem-based learning methods, so it is necessary that teachers be able to develop these on their own. Combining problem-based learning with thematic instruction should make the process easier, as the teacher has a common frame of reference to build upon.

For these two concepts, I have included two problem-based learning lessons: *How Many Fish Are In My Pond?* and *What's Wrong With My Pond?*. The specific goals and objectives of these lessons are listed in the included lesson plans. The higher level goal, in both cases, though, is to use a problem-based learning approach to help solidify the facts and skills learned during lecture-based learning lessons on these concepts. These lessons serve both as an instructional tool and as performance-based assessment tools. The teacher can assess how well the students have learned the facts and skills being taught by seeing whether they are applied correctly during the course of these lessons. Poor performance on these activities would be a sign to go back and spend more time teaching the facts and skills.

The *How Many Fish Are In My Pond?* lesson should elicit mathematical thinking and reasoning from the students based on the fact that they must apply what they have previously learned about experimental design and experiment simulation towards coming up with a suitable experiment for counting the fish. Since there is no single *right answer*, the students are free to explore whatever methods they desire, but since the goal is to design an experiment that will actually work, they must apply knowledge and reasoning to justify their proposals. Once the students decide on a method that uses the Lincoln-Petersen formula (or are guided there by the instructor), then they must apply mathematical thinking and reasoning to design the simulations. Actually getting to perform the simulations will then stimulate their thinking on whether they have come up with good simulations, what could be done to make either the experiment or the simulations better, and be able to make comparisons between the two different simulations.

The mathematical thinking and reasoning hoped for from the *What's Wrong With My Pond?* lesson is the ability to analyze available data and apply mathematical skills to make conclusions about the data. Since the lesson is designed to reinforce skills related to linear regression, then the students must think about and apply linear regression techniques. They must think about what the results of the linear regression means, and use reasoning to determine if they have found the cause of the declining fish population. This lesson would also be extremely useful for help students make connections between math and other subjects. For example, if it was known that students were studying wetlands ecology in a biology course, there might be a higher expectation that they would know what connections might exist between the fish and other living things in the area (e.g. that the perfect correlation between fish population and mosquito population was just a coincidence, since the fish are unrelated to the mosquitoes).

The *How Many Fish Are In My Pond?* lesson includes three different assessments. First, there is an "Additional Questions and Activity" sheet for the students to complete. This would be scored primarily as a "participation" grade, making sure that students attempted to provide thoughtful and reasoned answers to all the questions, and produced a suitable spreadsheet for presentation. There is a rubric which would be used by the instructor to assess performance on the overall activity. It could also be used by the students to perform a self-assessment. Finally, since there was a new skill learned during the course of the lesson (i.e. what the Lincoln-Petersen formula is and how to use it), there is a work sheet that can be used as a quiz.

During the *What's Wrong With My Pond?* lesson, the students are required to share the results of their linear regression calculations with the rest of the class. These results can be compared with the included answer key and used to assess their ability to correctly perform the linear regression process. In addition, the same rubric from the other lesson can be used with this lesson to assess the students' overall performance during the activity.

At the end of these lessons, students should be able to do more than just recite facts about experimental design and experiment simulation. They should be able to take a real world scenario and apply what they have learned towards creating a valid experiment, be able to determine if the experiment can be simulated, and if it can, be able to design a simulation using various methods. They should know how to perform linear regression, but more importantly they should understand how to apply it, how to decide if two data sets really are related, and how to communicate the linear regression results to others.

These lessons are designed to also serve as assessments, requiring the students to apply basic knowledge and skills learned in preceding lecture-based lessons. Those lecture-based lessons would be followed by traditional assessments such as quizzes and tests, composed primarily of closed-ended items, designed to assess their retention of facts and application of skills (e.g. to correctly find he line of best fit for given data set). Perhaps a few open-ended items would be included, increasing the complexity of the question and raising the students' thinking to a higher level. These lessons, then, attempt to bring the students' thinking and reasoning up to a more complex level, requiring a greater ability of abstract thought. Successfully completing these activities no longer requires a simple "plug and chug" performance by the students. Instead, they must demonstrate that they can think about and apply what they have learned, make connections between multiple concepts, and communicate what they know and the mathematics they can perform on a high level.